00:00:00,580 --> 00:00:25,830

[Music]

00:00:25,830 --> 00:00:25,840

Introduction

00:00:25,840 --> 00:00:28,160

Hi everyone I'm sony from London in the

00:00:28,160 --> 00:00:30,720

UK and today I have a really exciting

00:00:30,720 --> 00:00:37,190

video for you for the Virtual Maths

Camp (VMC)

00:00:37,190 --> 00:00:43,670

We're going to play a fun mathematical game called split or steal

00:00:43,670 --> 00:00:46,719

and this game is a two player game.

00:00:46,719 --> 00:00:49,840

So a quick introduction split or steal

00:00:49,840 --> 00:00:53,680

is a game based on the famous Prisoners Dilemma which you might have

00:00:53,680 --> 00:00:55,520

heard of before

00:00:55,520 --> 00:00:59,120

It's an interesting new branch of maths called Game Theory

00:00:59,120 --> 00:01:02,559

Where when making your choice you also need to consider

00:01:02,559 --> 00:01:05,439

The other player's choice

00:01:05,439 --> 00:01:10,950

This means your choices are interdependent

00:01:10,950 --> 00:01:14,080

So what you need today: a partner to play the game with,

00:01:14,080 --> 00:01:17,759

two small cards each saying split and steal,

00:01:17,759 --> 00:01:21,270

and these can just be small bits of paper

00:01:21,270 --> 00:01:23,680

and finally something to count scores with

00:01:23,680 --> 00:01:28,240

If you have some tokens or coins or something like that,

00:01:28,240 --> 00:01:29,990

That would be great,

00:01:29,990 --> 00:01:34,550

but if not you could just use a pen and paper to write down the

scores

00:01:34,550 --> 00:01:38,560

So this is what the game looks and this table is called a matrix

00:01:38,560 --> 00:01:46,720

In the game there are two points to be won but who wins what is decided by the choices of the players

00:01:46,720 --> 00:01:50,000

We have our two players on the left and at the top

00:01:50,000 --> 00:01:53,200

The red player and the blue player

00:01:53,200 --> 00:02:00,560

each player has two options shown next to them, split or steal

00:02:00,560 --> 00:02:07,040

Since two players have two choices each there are four outcomes in total

00:02:07,040 --> 00:02:10,239

and they are all shown in the table in each section

00:02:10,239 --> 00:02:13,760

The red number is the number of points won by the red player

00:02:13,760 --> 00:02:18,319

and the blue number is the number of points won by the blue player

00:02:18,319 --> 00:02:21,110

kwa mfano

00:02:21,110 --> 00:02:24,560

if both players choose to split

00:02:24,560 --> 00:02:28,400

we would end up with the top left outcome

00:02:28,400 --> 00:02:35,430

and the players would split the two points to earn one point each

00:02:35,430 --> 00:02:39,920

However if the red player wanted to split

00:02:39,920 --> 00:02:48,319

But the blue player chose to steal the blue player would steal the red player's point and earn two points

00:02:48,319 --> 00:02:52,879

while the red player wins nothing

00:02:52,879 --> 00:03:00,800

The opposite happens if the blue player splits and the red player steals

00:03:00,800 --> 00:03:06,080

but if both players try and steal

00:03:06,080 --> 00:03:14,560

it doesn't work and no one wins the points so both players end up with nothing

00:03:14,560 --> 00:03:21,910

Now it's your turn get ready to play split or steal with your partner

00:03:21,910 --> 00:03:27,190

First talk to your partner for a couple of minutes about what choice you're going to make

00:03:27,190 --> 00:03:33,750

Remember, the person with the most points wins

00:03:33,750 --> 00:03:37,040

You are allowed to lie to your partner then secretly choose split or steal

00:03:37,040 --> 00:03:42,959

and place the card you have chosen face down so your partner can't see it

00:03:42,959 --> 00:03:49,840

Finally reveal your choices and work out your scores

00:03:49,840 --> 00:03:57,350

Play the game once with your partner

00:03:57,350 --> 00:03:57,360

Pause the video now

00:03:57,360 --> 00:03:59,670

How did it go?

00:03:59,670 --> 00:04:01,599

Did you get the number of points you were hoping for?

00:04:01,599 --> 00:04:04,959

did you and your partner tell the truth to each other?

00:04:04,959 --> 00:04:12,390

Let's think about why the result might have been different to what you expected

00:04:12,390 --> 00:04:16,000

Imagine you are the red player your opponent

00:04:16,000 --> 00:04:25,510

The blue player has two choices split or steal

00:04:25,510 --> 00:04:25,520

if blue chooses split

00:04:25,520 --> 00:04:30,160

you could either choose split and win one point

00:04:30,160 --> 00:04:35,189

or you could choose steal and win two points

00:04:35,189 --> 00:04:41,189

two points is better than one so you would choose steal

00:04:41,189 --> 00:04:49,189

What if the blue player chose steal?

00:04:49,189 --> 00:04:53,440

If you choose split you get zero and if you choose steal you also get zero

00:04:53,440 --> 00:04:56,710

So it doesn't really matter what you choose

00:04:56,710 --> 00:05:03,990

But let's assume you prefer to steal so you don't give your opponent any points

00:05:03,990 --> 00:05:07,440

As we have seen no matter what your opponent does

00:05:07,440 --> 00:05:16,870

split is never the best choice this means steal is called a weakly dominant strategy

00:05:16,870 --> 00:05:27,830

Since this is a symmetric game steel is also weakly dominant for the blue player

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We have proven that it makes sense for both players to choose steal

00:05:34,870 --> 00:05:41,039

Therefore the steel steel outcome is known as the nash equilibrium

00:05:41,039 --> 00:05:54,800

but look at the matrix the split split outcome is better for both players as they both get one point instead of zero

00:05:54,800 --> 00:06:04,469

This means the nash equilibrium is not the optimal solution, an incredible result

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We now know what should happen for a single game

00:06:07,520 --> 00:06:13,990

But does this result hold if we play the game multiple times

against the same player

00:06:13,990 --> 00:06:18,400

Start the scoring from zero and play the game ten times in a row

00:06:18,400 --> 00:06:20,720

and play the game ten times in a row with your partner

00:06:20,720 --> 00:06:23,029

Does your strategy change?

00:06:23,029 --> 00:06:23,039

Now you know

00:06:23,039 --> 00:06:29,110

You'll be playing the same opponent again

00:06:29,110 --> 00:06:33,520

Play the game 10 times with the same partner

00:06:33,520 --> 00:06:39,270

Pause the video now

00:06:39,270 --> 00:06:39,280

[PAUSE]

00:06:39,280 --> 00:06:42,960

Did you manage to score more points than your opponent?

00:06:42,960 --> 00:06:47,680

A repeated game like the one you've just played is much more complicated

00:06:47,680 --> 00:06:52,240

Because your decision is not only influenced by your communication with your partner

00:06:52,240 --> 00:06:59,189

In this round but also what has happened in previous rounds

00:06:59,189 --> 00:06:59,199

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00:06:59,199 --> 00:07:04,629

You might trust your partner less if they stole in the previous round which could make you

00:07:04,629 --> 00:07:07,199

more likely to steal in this round

00:07:07,199 --> 00:07:10,230

In general,

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The more the game is repeated the more likely you will be to cooperate with your opponent

00:07:14,000 --> 00:07:18,400

Because they could punish you in future rounds if you don't

00:07:18,400 --> 00:07:22,400

Using the same logic, if you know there aren't many rounds left

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You might be more tempted to steal because your opponent has less time to retaliate

00:07:31,520 --> 00:07:36,479

As this is a very famous game game theorists have developed many strategies

00:07:36,479 --> 00:07:39,589

that we could use when playing

00:07:39,589 --> 00:07:39,599

kwa mfano

00:07:39,599 --> 00:07:44,479

You could always cooperate, meaning choosing split every time

00:07:44,479 --> 00:07:47,680

or you could play steal every time

00:07:47,680 --> 00:07:50,960

You might choose to copy what your opponent did in their last move

00:07:50,960 --> 00:07:55,199

sometimes known as tit for tat or copycat

00:07:55,199 --> 00:08:04,160

Grim trigger is where you play split but if your opponent plays steal just once you punish them by playing steal for the rest of the game

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You could even decide to choose randomly each time by flipping a coin

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Which strategy do you think is best?

00:08:15,670 --> 00:08:17,280

Try playing five rounds sticking to one of the strategies listed

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and see what happens if you can swap partners this time

00:08:21,919 --> 00:08:28,790

Then pick another strategy and play five more rounds

00:08:28,790 --> 00:08:32,399

play using a strategy

00:08:32,399 --> 00:08:37,269

Pause the video now

00:08:37,269 --> 00:08:37,279

[PAUSE]

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So, which strategy scored you the most points

00:08:41,200 --> 00:08:48,640

In 1980, Robert Axelrod made a tournament where he played 63 different strategies against each other

00:08:48,640 --> 00:08:53,360

To see which one came out on top

00:08:53,360 --> 00:08:58,630

and out of all of them it was tit for tat that won

00:08:58,630 --> 00:09:04,389

In general the most successful strategies were nice,

00:09:04,389 --> 00:09:09,350

Meaning they started off cooperating by playing split and forgiving

00:09:07,110 --> 00:09:11,040

Meaning that they wouldn't do what grim trigger does

00:09:11,040 --> 00:09:15,360

and fully stop cooperating once the opponent played steal

00:09:15,360 --> 00:09:23,760

I guess the fact that nice and forgiving strategies are the best

is a good sign for society

00:09:23,760 --> 00:09:27,670

and that's the end of this session

00:09:27,670 --> 00:09:30,790

if you enjoyed the topic, there's a very good website

00:09:30,790 --> 00:09:33,680

called nikki case's evolution of trust

00:09:33,680 --> 00:09:34,880

which goes into more detail

00:09:34,880 --> 00:09:41,920

Thank you and enjoy the rest of your Virtual Maths Camp.